AF TECHNICAL REPORT No. 6002 June 1950

FLOW DISTURBANCES DUE TO WALL IRREGULARITIES IN TWO-DIMENSIONAL SUPERSONIC FLOW

The Magnitude and Lateral Extent of Disturbance For Mach Numbers Between 1.05 and 1.25

Jean G. Goppert

Captain, USAF

United States Air Force
Air Materiel Command
Wright-Patterson Air Force Base, Dayton, Ohio

NOTE

When drawings, specifications, and other data prepared by the War Department are furnished to manufacturers and others for use in the manufacture or purchase of supplies, or for any other purpose, the Government assumes no responsibility nor obligation whatever; and the furnishing of said data by the War Department is not to be regarded by implication or otherwise, or in any manner licensing the holder, or conveying any rights or permission to manufacture, use, or sell any patented inventions that may in any way be related thereto.

The information furnished herewith is made available for study upon the understanding that the Government's proprietary interests in and relating thereto shall not be impaired. It is desired that the Patent & Royalties Section, Office of the Judge Advocate, Air Materiel Command, Wright-Patterson AFB, Dayton, Ohio, be promptly notified of any apparent conflict between the Government's proprietary interests and those of others.

Espionage Act

Notice: "This document contains information affecting the national defense of the United States within the meaning of the Espionage Act, 50 U. S. C., 31 and 32, as amended. Its transmission or the revelation of its contents in any manner to an unauthorized person is prohibited by law."

"The above Espionage Notice can be disregarded unless this document is plainly marked with a security classification as "Restricted," "Confidential," "Secret," or "Top Secret."

The U. S. Government is absolved from any litigation which may ensue from the contractor's infringing on the foreign patent rights which may be involved.

REPORT DOCUMENTATION PAGE	Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
JUNE 1950

3. REPORT TYPE AND DATES COVERED
FINAL JANUARY - JUNE 1950

4. TITLE AND SUBTITLE
FLOW DISTURBANCE DUE TO WALL IRREGULARITIES IN TWO-DIMENSIONAL SUPERSONIC FLOW: THE MAGNITUDE AND LATERAL EXTENT OF DISTURBANCE FOR MACH NUMBERS BETWEEN 1.05 AND 1.25

5. FUNDING NUMBERS

6. AUTHOR(S)
JEAN G. GOPPERT

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
AIRCRAFT LABORATORY
ENGINEERING DIVISION (WIND TUNNEL BRANCH)
AIR FORCE MATERIEL COMMAND
WRIGHT-PATTERSON AFB, OH 45433

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AIRCRAFT LABORATORY
ENGINEERING DIVISION (WIND TUNNEL BRANCH)
AIR FORCE MATERIEL COMMAND
WRIGHT-PATTERSON AFB, OH 45433

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AF-TR-6002

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
A METHOD IS PRESENTED FOR COMPUTING THE LATERAL EXTENT OF A WEAK SHOCK WAVE DUE TO A SMALL WEDGE-SHAPED IRREGULARITY OF A WALL IN TWO-DIMENSIONAL SUPERSONIC FLOW. COMPUTATIONS HAVE BEEN CARRIED OUT OVER THE MACH NUMBER RANGE 1.05 TO 1.25 FOR ANGULAR IRREGULARITIES OF THE WALL OF 0.1 TO 0.4 DEGREES. THE ENTROPY CHANGE IN THE SHOCK WAVE AS WELL AS THE INFLUENCE OF THE WALL BOUNDARY LAYER WERE ASSUMED TO BE NEGLIGIBLE.

14. SUBJECT TERMS
TWO DIMENSIONAL SUPERSONIC FLOW

15. NUMBER OF PAGES
15

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

Standard Form 298 (Rev. 2-89) (EG)
Prescribed by ANSI Std. 239.18
Designed using Perform Pro, WHS/DIOR, Oct 94

AF TECHNICAL REPORT NO. 6002 June 1950

FLOW DISTURBANCES DUE TO WALL IRREGULARITIES IN TWO-DIMENSIONAL SUPERSONIC FLOW

The Magnitude and Lateral Extent of Disturbance For Mach Numbers Between 1.05 and 1.25

Jean G. Goppert
Captain, USAF

United States Air Force
Air Materiel Command
Wright-Patterson Air Force Base, Dayton, Ohio

Air Force-WPAFB-(A)-O-27 JUL 50 100

FOREWORD

This report was prepared under Expenditure Order 903-1428 by Captain Jean G. Goppert, as project engineer in the Wind Tunnel Branch, at the request of the Chief of the Project Unit, Wind Tunnel Branch.

Included among those who cooperated in reviewing and editing the report were Dr. B. H. Goethert and Mr. R. R. Kassner of the Wind Tunnel Branch.

ABSTRACT

A method is presented for computing the lateral extent of a weak shock wave due to a small wedge-shaped irregularity of a wall in two-dimensional supersonic flow. Computations have been carried out over the Mach number range 1.05 to 1.25 for angular irregularities of the wall of 0.1 to 0.4 degrees. The entropy change in the shock wave as well as the influence of the wall boundary layer were assumed to be negligible. Computations were carried out for the straight part of the shock wave and for such points along the curved part of the shock wave at which the local Mach number change, ΔM, across the shock wave, is reduced to values of 0.001, 0.002, 0.003, 0.004, and 0.005.

PUBLICATION REVIEW

Manuscript Copy of this report has been reviewed and found satisfactory for publication.

FOR THE COMMANDING GENERAL:

Jack A. Gibbs

JACK A. GIBBS
Colonel, USAF
Chief, Aircraft Laboratory
Engineering Division

TABLE OF CONTENTS

Page

SYMBOLS

M Mach number

M_1 Free stream Mach number

M_2 $(M_1 - \Delta M_2)$ Mach number behind the shock wave, but in front of the expansion fan

M_3 $(M_1 - \Delta M_3)$ Mach number along some expansion wave at which the flow has attained a Mach number arbitrarily close to M_1.

ℓ Length of the wedge

δ Wedge angle

Δh Maximum wedge thickness

θ Angle measured counterclockwise from the horizontal

θ_1 Mach angle of flow at M_1

θ_2 Angle of first expansion wave

θ_3 Angle of the expansion wave along which the flow has attained the Mach number M_3

ϵ Angle of the shock wave at any point

ϵ_2 Angle of the shock wave before intersection with the expansion fan

α Mach angle measured from the local flow direction

ν_1 Deflection angle required to change the flow Mach number from $M = 1.0$ to M_1 in a Meyer expansion

ν Deflection angle required to change the flow Mach number from $M = 1.0$ to M_2

r Radius from the rear corner of the wedge

r_2 Radius to the point of intersection of the shock wave and the first expansion wave

r_3 Radius to the point on the shock wave where the Mach number behind the shock has attained the value M_3

y Lateral distance (perpendicular to free stream)

y_2 Distance from wall to point of intersection of shock wave and first expansion wave

y_3 Distance from wall to point along shock wave where the Mach number behind the shock has attained the value M_3

$\gamma = \frac{C_p}{C_v}$ Ratio of specific heats

SECTION I

PURPOSE

The purpose of this investigation is to formulate an estimate within practical accuracy of the magnitude and extent of the disturbance due to a small irregularity of a wall in two dimensional supersonic flow. Consideration is limited to low supersonic Mach numbers and small irregularities, for which only attached shock waves appear and the shock losses are negligible. Boundary layer influences are not included, but it can be expected that they will weaken the intensity of the flow disturbance.

SECTION II

DERIVATION

The problem has been simplified by considering the special case of a plane surface with a wedge shaped irregularity in supersonic flow.

Figure I shows the wedge shaped irregularity AB. Upstream of A and downstream of B the wall is assumed to be plane and parallel to the undisturbed flow. A shock wave originates at A, changing the flow angle from 0 to δ and the Mach number from M_1 to M_2. These flow conditions, however, exist only within the triangle ABP_2. At point P_2 (wall distance Y_2) the shock wave is intersected by the first of the expansion waves which start from the corner at B. Beyond P_2 the shock wave is curved due to intersection with the expansion waves and the intensity of the shock wave is reduced. Therefore, the Mach number change, ΔM_3, $(M_1 - M_3)$, across the shock wave, occurring at a point, P_3 (wall distance Y_3), on the curved part, is smaller than the change ΔM_2 across the straight part of the shock.

The object of the following computation is to determine:

a. Mach number disturbance, ΔM_3, across the curved part of the shock as a function of distance Y_3.

b. Mach number disturbance across the straight part (equal to disturbance at wall) and distance Y_2.

Curved Part of the Shock Wave

Figure 2 shows an element of the shock wave between two expansion waves having the angles θ and $\theta - d\theta$ with the free stream direction. The shock wave element forms the angle ϵ with the free stream direction.

Therefore, since $(\theta - \epsilon)$ is the angle between the shock wave element and an expansion wave, and r is the radius from the corner, B, we have:

$$\frac{dr}{-rd\theta} = \cot(\theta_2 - \epsilon) \qquad (1)$$

integrating from r_2 to r_3 (See Figure 1)

$$\ln \frac{r_3}{r_2} = \int_{\theta_3}^{\theta_2} \cot(\theta - \epsilon)\, d\theta \qquad (2)$$

Thus the first problem is to set up $(\theta - \epsilon)$ in terms of θ. For a given initial Mach number, M_1, and for assumed values of the shock wave angle, ϵ, the Mach number, M, behind the shock wave can be computed from the equation (Eq. 4.33, Ref. 3).

$$M^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 \sin^2\epsilon - \frac{\gamma-1}{2}} + \frac{M_1^2 \cos^2\epsilon}{1 + \frac{\gamma-1}{2} M_1^2 \sin^2\epsilon} \qquad (3)$$

The proper range of assumed values of ϵ was determined from a plot of δ vs ϵ for constant values of M_1, taken from Table I of Ref. 2. By neglecting the entropy changes in the weak oblique shocks, these computed values of M, together with Table II of Ref. 1, for issentropic flow, can be used to obtain θ, which is the angle of the expansion wave along which the Mach number is M. That is:

$$\theta = \alpha + (\nu_1 - \nu) \qquad (4)$$

where α is the Mach angle corresponding to the Mach number, M, and $(\nu_1 - \nu)$ is the flow inclination along the Mach wave with respect to the direction of the undisturbed flow with $M = M_1$. (See Figure 3.)

Thus we have calculated values of θ and ϵ corresponding to various values of M. It is now possible to compute and plot $(\theta - \epsilon)$ vs θ.

If the plot of $(\theta - \epsilon)$ vs θ can be approximated by a straight line over the desired range of θ, then the integral (Eq. 2) can be evaluated very easily since we can substitute

$$\theta - \epsilon = \frac{\theta + c}{a} \qquad (5)$$

where the constants a and c can be evaluated from the geometry of the line as

$$a = \frac{d\theta}{d(\theta - \epsilon)}$$

$$c = -\theta_1$$

Similarly, a slight curvature of the plot $(\theta - \epsilon)$ vs θ can be approximated by two straight lines and integrated in two intervals. If, however, the curvature is too great, then a graphical integration seems to be the best procedure. Merely plot a curve of cot $(\theta - \epsilon)$ vs θ, compute the area under the curve between the limits θ_2 and θ_3, and take the antilog to get $\frac{r_3}{r_2}$.

The angle θ_2 is defined as the angle of the first expansion wave of the Prandtl-Meyer expansion, (Figures 1 and 3).

Thus
$$\theta_2 = \alpha_2 + \delta \tag{6}$$

θ_3 is defined as the angle of the expansion wave along which the flow has reached any arbitrary Mach number, M_3, which is determined by the choice of a value of ΔM_3 across the shock wave. After a selection of ΔM_3 is made, θ_3 can be computed by means of Eq. 3 and 4.

Straight Part of the Shock Wave

The next step is to compute the value of $\frac{r_2}{l}$ which is the radius from the expansion corner, B, to the point of the intersection of the shock wave with the first expansion wave, expressed in terms of the length of the wedge. This is a simple problem in trigonometry since the initial shock wave angle, ϵ_2, and the Mach number behind the shock, M_2, can be determined from tables in Ref. 2, Eq. 3. Since the direction of a Mach wave is given by the Mach angle, which depends upon the known Mach number of the flow, we can, by referring to Figure 1, see that the law of sines will give the desired radius, $\frac{r_2}{l}$:

$$(1)\quad \frac{r_2}{l} = \frac{1}{\cos\delta} \cdot \frac{\sin(\epsilon_2 - \delta)}{\sin(\theta_2 - \epsilon_2)} \tag{7}$$

$$(2)\quad \theta_2 - \epsilon_2 = \alpha_2 - \epsilon_2 + \delta$$

$$(3)\quad \frac{r_2}{l} = \frac{\sin(\epsilon_2 - \delta)}{\cos\delta\,\sin(\alpha_2 - \epsilon_2 + \delta)}$$

It is now a simple step to determine $\frac{Y_3}{l}$ (See Figure 1)

$$\frac{Y_3}{l} = \frac{r_3}{l}\sin\theta_3$$

RESULTS AND DISCUSSION

Computations were carried out for the Mach number range 1.05 to 1.25, and for wedge angles of 0.1, 0.2, 0.3, and 0.4 degrees, using arbitrary values of ΔM_3 = .001 to .005. The results are presented in Figures 4 and 5.

The probable error in Y_2 and Y_3 may be as great as 5% near $M_1 = 1.05$, with considerable improvement as the Mach number increases. This large error is due to the extremely small difference between the shock wave angle and the angle of the expansion waves. However, it must be remembered that only the approximate order of magnitude of the lateral extent of the disturbance is all that was to be obtained from this calculation.

The case of negative values of the wedge angle, δ, was checked at $M_1 = 1.15$. The values of $\frac{Y_3}{\ell}$ were found to be of the same order of magnitude but slightly smaller than the values for positive angles. The difference was so small, however, that further computations did not appear to be warranted.

The results have been presented in Figure 5 as a single family of curves of ΔM_3 vs $Y_3/\ell\delta$ for lines of constant Mach number, M_1. The computations showed that Y_3/ℓ was proportional to δ within the investigated range of wedge angles and Mach numbers and within the accuracy of these computations. It should be noted, also, that the values given in Figure 5 are not valid for values of $Y_3 < Y_2$, (see Figure 1), since the disturbance ΔM is constant from $Y = 0$ to $Y = Y_2$.

The distance, Y_2, as well as the disturbance ΔM_2, valid for $0 \leqq Y \leqq Y_2$ has been plotted in Figure 4 with the wedge angle δ as a parameter. For the purpose of interpolating between the curves, it may be remarked that at constant Mach number, M_1, ΔM_2 is approximately proportional to δ, while $Y_2/\Delta h$ is approximately proportional to $1/\delta^2$. Furthermore, for a given wedge, the distance Y_2 varies approximately with the Mach number according to $\sqrt{M_1^2 - 1}$.

APPLICATION OF RESULTS

There are two problems which can be solved quite easily using Figure 4.

a. Problem 1. Given a wall configuration, δ and ℓ, and a free stream Mach number, M_1. Find the strength of the disturbance, ΔM_3, at some given distance, Y_3, from the wall.

Solution. Compute $\frac{Y_3}{\ell \cdot \delta}$, and read the value of ΔM_3 from the curve on Figure 5 corresponding to the proper value of M_1.

b. Problem 2. Given the maximum allowable disturbance, ΔM_3, at a distance, Y_3, from the wall, and a free stream Mach number, M_1. Find the allowable height of wall disturbance $\Delta h = \ell\,\delta$ which would satisfy the maximum allowable disturbance requirements.

Solution. From Figure 5, find the value of $\frac{Y_3}{\ell \cdot \delta}$ corresponding to the given values of M_1 and ΔM_3. Then, since Y_3 is given, this determines the allowable disturbance height, $\ell \cdot \delta$.

If, in the problem to be solved, the wall has a curved irregularity instead of the assumed abrupt wedge-shaped irregularity, the results given in this report will be conservative since the initial shock wave will be split up into a number of compression waves which will probably converge into a shock wave, but the intensity will be less than for the wedge.

The same effect could be expected in considering the boundary layer, since the boundary layer would tend to round off the corners of the wedge.

SUMMARY

The local Mach number disturbances in two dimensional flow, due to a wedge shaped wall irregularity, are calculated for various lateral distances from the wall for the Mach number range between $M = 1.05$ and 1.25. (See Figures 4 and 5.)

When the height of the disturbance $\Delta h = \delta \cdot \ell$ is kept constant, the lateral extent of the region between the initial shock wave and the first expansion wave increases inversely with the square of the wedge angle of the wall irregularity and varies with the Mach number according to $\sqrt{M_1^2 - 1}$. (See Figure 4.)

In the region behind the first intersection of initial shock wave and expansion wave, the local Mach number disturbance is nearly independent of Mach number and wedge angle, but rather depends mainly on the ratio of the lateral distances to the disturbance height. (See Figure 5.)

REFERENCES

1. NACA TN 1428 - Notes and Tables for Use in the Analysis of Supersonic Flow. December 1947

2. NACA TN 1673 - Tables and Charts of Flow Parameters Across Oblique Shocks, by Mary M. Neice, August 1948

3. Liepmann & Puckett - "Aerodynamics of a Compressible Fluid" 1947

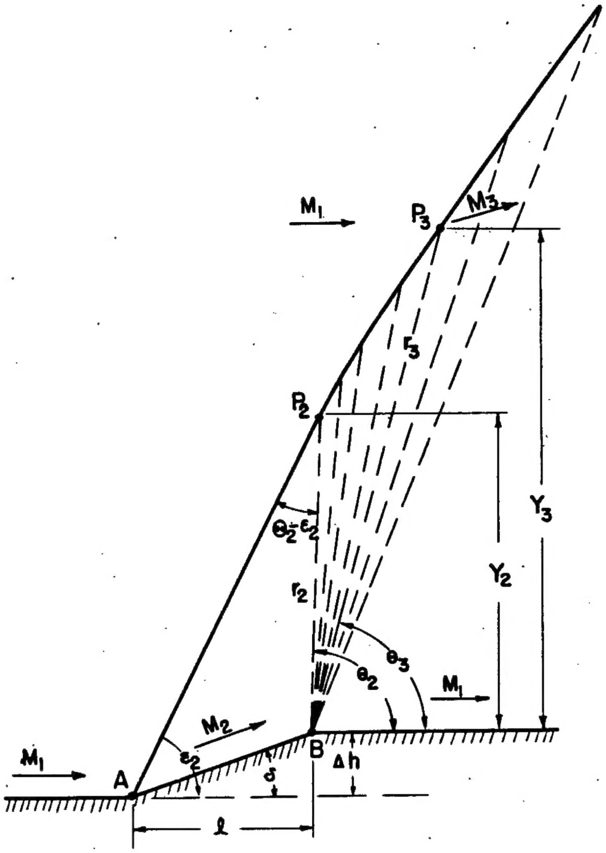

Figure 1: Diagram of Shock Wave Pattern Due to Wedge-Shaped Irregularity

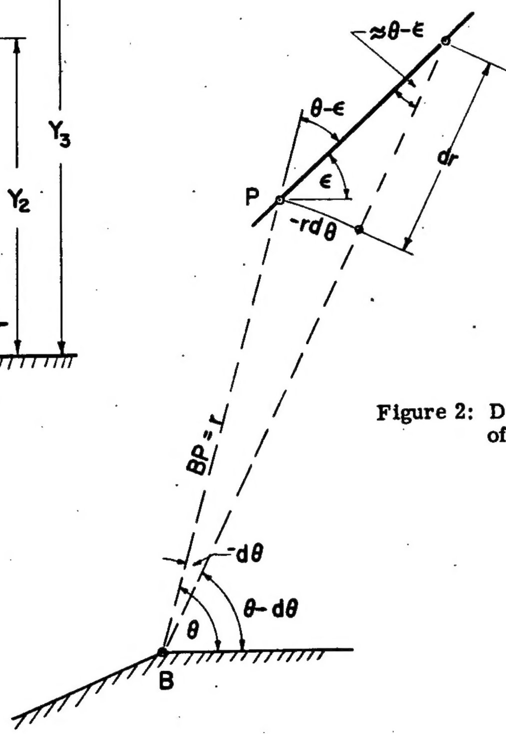

Figure 2: Detail Sketch of Element of Shock Wave

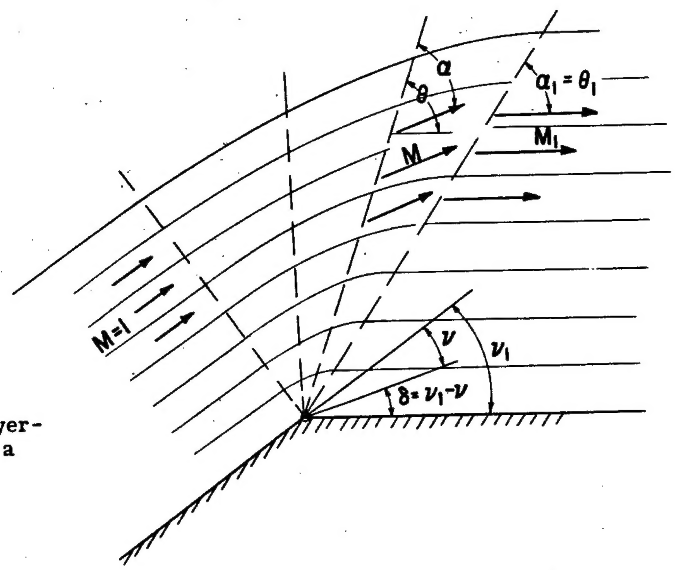

Figure 3: Definition of Angles in Meyer-Prandtl Expansion Around a Corner

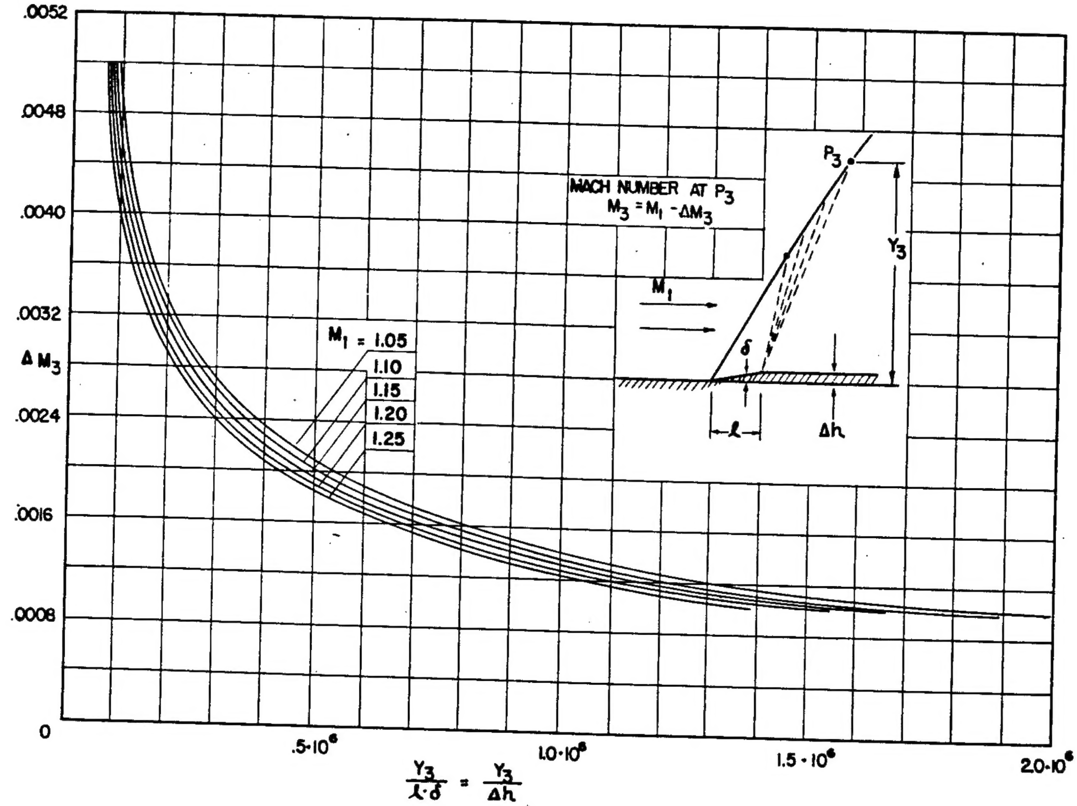

Figure 4: Lateral distance to first intersection of shock wave and expansion fan and Mach number disturbance at the wall for small wedge-shaped irregularity of a two-dimensional wall.

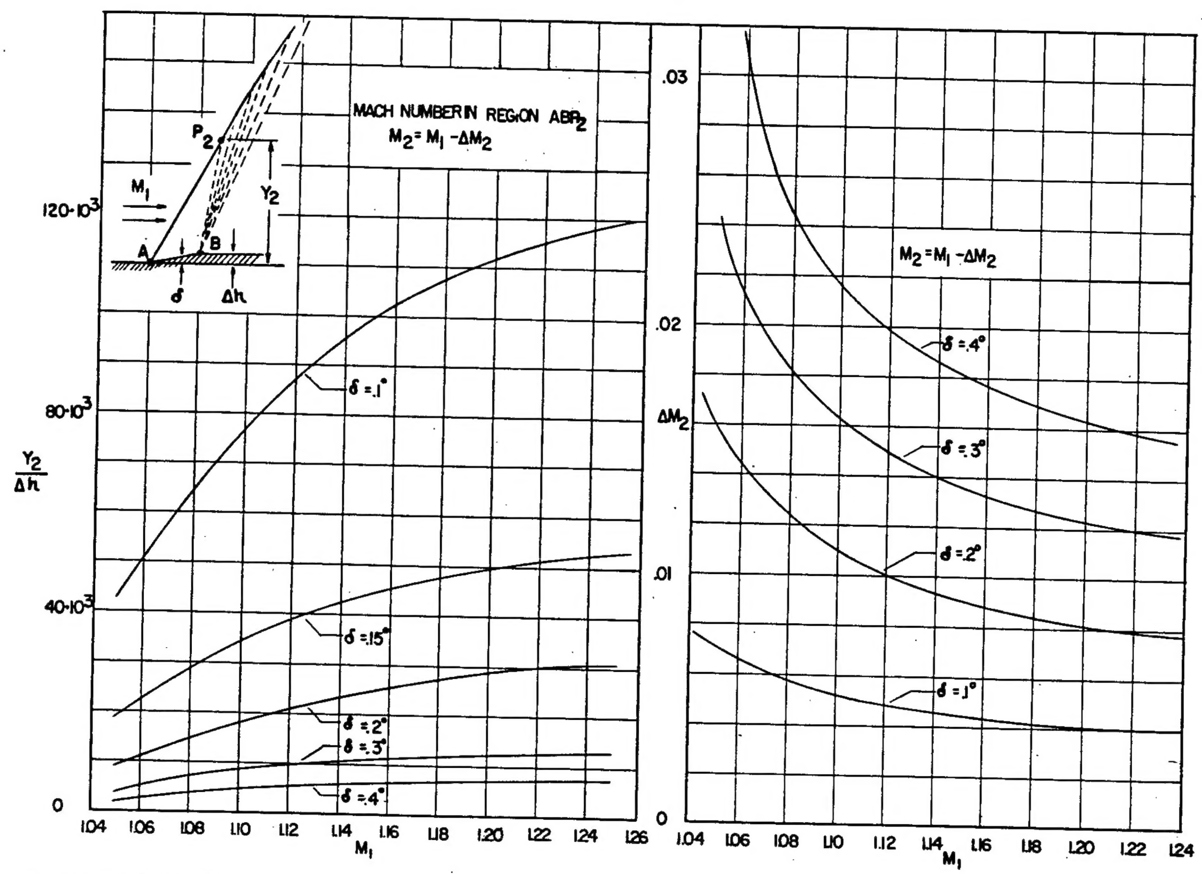

Figure 5: Lateral variation of Mach number disturbance due to a small wedge-shaped irregularity of a two-dimensional wall. Region downstream of first intersection between shock wave and expansion wave.